



Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at <http://about.jstor.org/participate-jstor/individuals/early-journal-content>.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

THE ORIGIN OF LETTERS AND NUMERALS ACCORDING TO THE SEFER YESIRAH

BY PHINEAS MORDELL, Philadelphia

THERE is hardly another book in Jewish literature, the Bible and the Talmud excepted, that has been so much commented upon as the Sefer Yeşirah. It has been the subject of deep study, not only to the mystic, who regarded it as the source of esoteric lore, but also to the philosopher and the Talmudist. And yet, despite all the efforts of a large number of scholars of repute for more than a thousand years, the Sefer Yeşirah remains a sealed book. The various commentaries upon it are more apt to bewilder the student than to enlighten him. Indeed, it would seem that every commentator endeavored to read his own views and theories into this little book, with hardly any concern whether they agreed with the text or not.

There is no book in Jewish literature that is so difficult to understand as the Sefer Yeşirah. For it was originally written in an obscure half-mystical style. To make matters worse, the commentators of the eighth or of the ninth century blended the original "Sefer Yesirah" with an early commentary, which may be referred to as "Sefer Yesirah II." It thus happened that all the commentaries written on the "Sefer Yesirah" since the beginning of the tenth century are chiefly based on this commentary and not on the original Sefer Yesirah. Although the Sefer Yesirah is exceedingly hard to understand, the solution of its many difficul-

ties is not impossible. The reason why they have remained so long an unsolved problem is partly due to a lack of knowledge of Hebrew orthography, on which the *Sefer Yeşirah* is based. In spite of the numerous works written on Hebrew orthography since the beginning of the tenth century, there is not one which may be considered as really based on the Hebrew. For the Hebrew orthography which has been and is still taught, is not Hebrew but Arabic. The Hebrew grammarians under Arabic influence came to believe that those rules of orthography which the Arab grammarians discovered for the language of the Koran hold good also for the language of the Old Testament. When the Honorable Mayer Sulzberger heard me expressing my views on Hebrew orthography, he advised me to make a study of the *Sefer Yeşirah* which in his opinion constitutes the earliest Hebrew grammar extant. Finding that my views on Hebrew orthography harmonized with those of the *Sefer Yeşirah*, I made an exhaustive study of it. After many years of study, I reached the conclusion that the *Sefer Yeşirah*, as the earliest Hebrew grammar, contains not only the fundamental rules of Hebrew orthography, but also an account of the origin of letters and numerals. This account it is my present purpose to set forth.¹

The first Mishnah reads as follows: שלשים ושנים נתיבות : נתיבות פלאות חכמה חקק יי יהוה צבאות בסופר וספור וספּר.

"Thirty-two mysterious ways of wisdom has the Lord, the Lord of hosts, ordained through Scribe, Script, and Scroll."

The thirty-two ways of wisdom are the twenty-two

¹ I am indebted to Dr. I. Broyde, Miss Henrietta Szold, and Dr. Isaac Husik who have given me valuable aid and suggestions.

letters of the Hebrew alphabet, which represent thirty-two sounds.²

In accordance with the belief of the ancients that the letters are of divine origin, the Sefer Yeṣirah explains that the thirty-two ways of wisdom were ordained by God through:

(1) סופר (Scribe), the man whom God inspired to invent the alphabet;

(2) ספוד (Script), the letters;

(3) ספר (Scroll), the material on which the letters were displayed.

In order to show how the twenty-two letters of the alphabet constitute the thirty-two ways of wisdom, the author proceeds to the division of the letters in the second and following Mishnahs. He divides them into simple and double sounds, and also into vowels and consonants. The simple letters he called פשוטות, and the double, כפולות; the vowels, אמות (=אבות), and the consonants, תולדות. Altogether they form thirty-two sounds: the twenty sounds of the ten³ double letters, and the twelve of the twelve simple letters.

The Sefer Yeṣirah⁴ emphasizes that the number of the double letters is no less and no more than ten, and the number of the simple letters no less and no more than twelve. The Sefer Yeṣirah urges us to investigate and examine the letters, that we may have a clear insight into

² All the commentators explain that the thirty-two ways of wisdom are the twenty-two letters and the ten Sefirot. Below will be found the reasons why the present writer cannot accept this interpretation.

³ Below will be explained that, according to the Sefer Yeṣirah, there are ten double letters and not only seven, as is believed by all commentators since Saadya.

⁴ See text, § 3.

the subject. This proves that, at the time when the book was written, the nature of the letters, or of some of them, was misunderstood. We know, indeed, that at the time when the Greek translation of the Bible was made, it was believed that the *ṣ*, for example, could be transliterated by *e*, *a*, or *g*, and the translators accordingly rendered it variously by one of these three sounds.

Arguments have repeatedly been advanced in favor of the view that the Hebrew *ṣ* had not only the sound of the Arabic *‘*, but also of the *ġ*. But according to the *Sefer Yeṣirah*, the *ṣ* is a simple letter. If it has the sound of *‘* it is impossible that it should also have the sound of *ġ*. Moreover, if the *ṣ* was originally a vowel only and had no sound of *g*, as maintained by Jerome, it can have only one vowel sound. If we ascribe to it the sound of *A*, it is impossible that it should have also the sound of *E* or *O*, etc. Furthermore, according to the *Sefer Yeṣirah*, the letters *ḥ*, *ḳ*, *ṣ* are also simple letters, and each must have had only one sound and not two as in Arabic.

The author of the *Sefer Yeṣirah* apparently cautioned against the very errors and mistakes into which all writers on Hebrew grammar have fallen. By dividing the twenty-two Hebrew letters into ten double and twelve simple, representing thirty-two sounds, the author desired to make clear how different the Hebrew alphabet is from the alphabet which is known as Arabic and which the Arabs themselves used to call *sūrī*. By *sūrī* was apparently meant Assyrian. The so-called Arabic alphabet consisted originally of only seventeen letters. It was apparently originally invented to represent the Assyrian-Babylonian language, which consisted of seventeen or eighteen sounds. Hence the name "*sūrī*." When the Arabs, whose original alphabet

was the Himyarite, consisting of twenty-eight letters, adopted the "*sūrī*" alphabet, they gave to some "*sūrī*" letters two or even three sounds, and such letters are each counted now as two or three letters.

As according to the Sefer Yeṣirah the Hebrew alphabet consists of ten double letters and twelve simple, therefore to each double letter must be ascribed two sounds and to each simple letter only one sound without any regard to their value in Arabic.

The best transliteration of the double letters is as follows: א = *e*, א = *o*, ב = *b*, ב = *v*, ג = *g*, ג = as English *j* (?), ד = *d*, ד = *th* in "*the*," ו = *u*, ו = *v*, כ = *k*, כ = German *ch* or Arabic *ḥ*, פ = *p*, פ = *f*, or *ph*, ר = *r*, ר = *r* or Arabic *ḡ*, ש = *sh*, ש = French *j* (?), ת = *t*, ת = *th* in "*think*."

The best transliteration of the simple letters is as follows: ה = *h*, ז = English *z*, ח = *h* (Arabic *ḥ*), ט = *t*, י = German *j*, ל = *l*, מ = *m*, נ = *n*, ס = *s*, ע = *a*, צ = *tz*, ק = *k*.

After giving this division of the letters, the Sefer Yeṣirah shows how, from these twenty-two letters, all the words that have ever existed or ever will exist can be formed.⁵ The purpose of the author was to emphasize the superiority of alphabetic writing over the non-alphabetic writing (ideographic and syllabic) used by all the nations of antiquity, and even now by a great portion of mankind. If we arrange alphabetically all biliteral combinations, as the Sefer Yeṣirah directs, joining א with all letters, ב with all letters, etc., there must result a list of 484 combinations ($22^2 = 484$). (See next page.)

⁵ See text, §§ 5, 6, and 7.

[illegible]

Furthermore, by the expression וַחֲזָרָה הַגְּלִיל פְּנִים וְאַחֲרָה the Sefer Yeširah indicates that the biliteral combinations can be made the basis of all trilateral combinations. If we desire to arrange all the trilateral combinations that can be formed from the 22 letters, their number will be 22^3 or 10,648. For this it would be necessary to draw up twenty-two tables with the biliteral combinations, leaving sufficient space between every two combinations for the addition of a letter. On one table an א would have to be added at the beginning of each biliteral combination, and the result would be a complete table of 484 trilateral combinations beginning with an א; on another a ב would be added in the same way, making a complete table of 484 trilateral combinations with the letter ב at the beginning. Proceeding thus with the remaining letters, we should get all possible trilateral combinations that can be made out of the twenty-two letters. In this way two-thirds of the labor otherwise necessary is saved, for adding the third letter is only one-third of the labor required to produce all the trilateral combinations. Should we desire to write all the quadriliteral combinations that can be made out of the 22 letters, we have only to make twenty-two copies of all the trilateral combinations, leaving sufficient space between two successive combinations for the addition of a new letter; then by adding an א at the beginning of each trilateral combination, we shall attain 10,648 quadrilaterals beginning with an א. Proceeding in the same way with ב, we shall obtain 10,648 quadrilaterals beginning with ב, and so forth with the remaining letters, which would give a total of 22^4 , or 234,256. The number of quinqueliteral combinations would amount to 22^5 , or 5,153,632.

The powers of twenty-two up to 12 are as follows :

484	$= 22^2$
10,648	22^3
234,256	22^4
5,153,632	22^5
113,379,904	22^6
2,494,357,888	22^7
54,875,873,536	22^8
1,207,269,217,792	22^9
26,559,922,791,424	22^{10}
584,318,501,411,328	22^{11}
12,855,002,631,049,216	22^{12}

Thus it is evident that the twenty-two letters will admit of an infinity of combinations and arrangements, sufficient to represent not only all conceptions of the mind, but all words in all languages whatever.

The same results would be obtained, according to the Sefer Yeşirah, by adding a letter at the end of each combination. When a letter is added at the beginning, the process is called וְחֹרֶר הַגִּלְגָּל פָּנִים, the table "turns" in front of each letter, as נג "turning" in front of the ע becomes ענג, and when a letter is added at the end, it is called וְחֹרֶר הַגִּלְגָּל אַחֲרֵי, the table "turns" behind each letters, as נג turning behind the ע becomes נגע. Thus, as either ענג or נגע can be formed from the combination by adding an ע, so all the trilateral combinations can be made out of the biliteral combinations, by adding an additional letter, either at the beginning or at the end of the biliteral combinations, and the quadrilaterals from the trilaterals, etc., without the necessity of writing the letters anew, when new combinations are desired.

How infinite numbers of words are formed out of the twenty-two letters, the Sefer Yeṣirah demonstrates by permutations in which letters never repeat themselves but only change their places. Out of two letters⁶ two biliteral words are formed as: **אב, בא**. Out of three letters six triliteral words are formed as: **אבג, אגב, באג, בגא, גאב, גבא**. Out of four letters 24 quadriliteral words are formed as: **אבגד, אבדג, אגבד, אגדב, באגד, באדג, גאבד, גאדב, דאבג, דאגב, דבאג, דבגא, דגאב, דגבא, גאדב, גבאד, גבדא, גדאב, גדבא, דאגב, דאבג, דבגא, דגבא**. Out of five letters 120 quinqueliteral words are formed, out of six letters 720 six-letter words are formed and out of seven letters 720 six-letter words are formed and out of seven letters 5,040 seven-letter words are formed. The Sefer Yeṣirah gives the factorials up to that of seven and concludes the Mishnah by saying: "Go and count further, what the mouth is unable to pronounce, and the ear is unable to hear."

The factorials up to that of 12 are as follows:

1	=	(1
2	=	(2
6	=	(3
24	=	(4
120	=	(5
720	=	(6
5,040	=	(7
40,320	=	(8
362,880	=	(9
3,628,800	=	(10
39,916,800	=	(11
479,001,600	=	(12

⁶ See text, § 8.

The factorials up to 36 are given in Rees' Encyclopedia, art. *Cipher*. The Mishnah treating of permutation was well explained by all commentators, especially by S. Donolo.

W. Stanley Jevons on the subject of permutation says: "Thus the letters *A, B, C*, will make different permutations according as *A* stands first, second or third; having decided the place of *A*, there are two places between which we may choose for *B*; and then remains but one place for *C*. Accordingly, the permutation of these letters will be altogether $3 \times 2 \times 1$ or 6 in number. With four things or letters *A, B, C*, and *D*, we shall have four choices of places for the first letter, three for the second, two for the third, and one for the fourth, so that there will be altogether $4 \times 3 \times 2 \times 1$, or 24 permutations. The same simple rule applies to all cases; beginning with the whole number of things, we multiply at each step by a number decreased by a unit"

He further says: "Many writers have from time to time remarked upon the extraordinary magnitude of the numbers with which we deal in this subject. Taquet calculated that the twenty-four letters of the alphabet may be arranged in more than 620 thousand trillions of orders; and Schott estimated that if a thousand million of men were employed for the same number of years in writing out these arrangements, and each man filled, each day, forty pages with forty arrangements in each, they would not have accomplished the task, as they would have written only 584 thousand trillions instead of 620 thousand trillions."

[†] The Principles of Science, London 1887, 178, 179.

THE TETRAGRAMMATON AND THE VOWELS

All words arising from the combination of the letters are combined and permuted with the Tetragrammaton, called by the Sefer Yeṣirah "One Name" יהוה, from which emanated the whole of creation and all languages.⁸ According to the Sefer Yeṣirah, the alphabet did not consist of consonants only, as is held by many Semitic scholars, but had vowels also, the letters of the Tetragrammaton themselves being vowels. Indeed, the ancients transcribed יהוה by the vowels *i-e-u-o*, and *i-a-o-u-i*.⁹

To understand how the letters of the Tetragrammaton can be vowels, it is necessary to know what the Hebrew vowels are. Here the view of various writers differ widely. Dunash Ibn Tamim,¹⁰ whose opinion was shared by many Hebrew writers, maintained that the three letters איו are the original vowels of the Hebrew alphabet; Roger Bacon,¹¹ Masklef, and others held that the six letters אהווע were originally vowels; and Jerome and many others asserted that the five letters אהויע were the original vowels. In my opinion the original vowels are the four letters אעיו, which are still used as vowels in transcribing other languages in Hebrew characters. The Sefardic Jews, when writing Spanish with Hebrew letters, transcribe *a* by א; *i* and *e* by י; and *u* and *o* by ו. The Ashkenaztic Jews, when writing German with Hebrew letters, transcribe *a* and *o* by א, *e* by ע, *u* by ו, and *i* or *j* by י. An investigation into the relation of the letters to the vowel points according to the Ashkenazic pronunciation

⁸ See text, § 9.

⁹ Comp. Renan, *History of the People of Israel*, Boston 1888, I, 69.

¹⁰ Sefer Yeṣirah, London 1902, 20, 45, 48.

¹¹ See *JQR.*, XV, 336.

led me to the conclusion that γ has the sound of *a*, \aleph of *e*, and *o*, ι of German *j*, and υ of *u*, besides its sound of *w*.

If this opinion with regard to the original letters be correct, not only an \aleph is hidden under a η of the Tetragrammaton יהוה, as believed by Dunash Ibn Tamim,¹² Judah ha-Levi, and Abraham Ibn Ezra, but also an γ . In order to understand the secret of the Tetragrammaton, the nature of the η must be better defined. We have seen that many authors since Jerome believed η to be a vowel. This is not the opinion of the Sefer Yeşirah. It counts η among the simple letters, and, consequently, no sound in addition to that of *h* can be attached to it. On the other hand, the η occurs at the end of words as silent, indicating that it occupies the place of a vowel letter. We must therefore assume that the Sefer Yeşirah considers η to play the same part among the vowel letters as zero does among numerals. As zero is not a digit itself, but only occupies the place of a digit, so the silent η is not a vowel itself, but merely occupies the place of a vowel. In the Tetragrammaton, both the η after the υ and the η after the ι occupy the place of a vowel letter. The original letters of the Tetragrammaton were יעוה instead of יהוה.

Now, what is the "great secret," which the three mother letters (vowels) אמש contain according to the Sefer Yeşirah? Dunash Ibn Tamim declared it to be the three vowel letters אוי, which he identified with the Tetragrammaton יהוה the "secret name." In my opinion, אמש stands for אעו, and אמת which occurred in the Rashi text of the Sefer Yeşirah (Rashi, commentary on Job 28, 27; S. Sachs, הייתה, p. 94) stands for אעי. According to the original Sefer Yeşirah there are four mother letters, אמשת, crypto-

¹² Sefer Yeşirah, London 1902, 45.

grams which should be deciphered by the letters אעיו: the א by א, the מ by the ע, the ש by the ו, and the ת by the י. From these four letters emanated, according to the Sefer Yeṣirah, the four elements, air, water, fire, and earth.

The Sefer Yeṣirah apparently used אמת as secret characters representing אעיו, because the Hebrew alphabet is arranged with the מ at the beginning, the ש in the middle, and the ת at the end; while the Arabic alphabet was originally arranged with the א at the beginning, the ע in the middle, and the ו and י, at the end. The Arabic alphabet consisted originally of the following seventeen letters:

א ב ג ד ر س ص ط ع ف ك ل م ن ه و ي
 א ב ג ד ר ש צ ט ע פ כ ל מ נ ה ו י

The fact that the Arabic alphabet has the א at the beginning, and the ע in the middle, and the ו and the י at the end, leads to the conclusion that the arrangement of the Arabic alphabet is older than that of the Hebrew, although the contrary is generally believed to be the case.

The alphabet was anciently believed to symbolize the whole universe. The vowels which were the original letters of the Tetragrammaton, were placed at the beginning, in the middle, and at the end of the alphabet, to signify that יהוה is the God of the whole universe, that He is the beginning, the middle, and the end.

According to their sounds, we may count, in the Hebrew alphabet, thirty-two letters, divided into five vowels, (ע = a, א = e, ו = o, ו = u, י = i) and twenty-seven consonants. Each vowel, according to the Sefer Yeṣirah (see text, § 12), stands by itself, but the consonants are dependent on the vowels. The vowels and consonants were made in the form of a “state and arranged like an

army in battle array." Isaac Taylor¹³ says: "Like soldiers on parade the characters in the alphabetic line have been dressed."

SEFIROT

The origin of the alphabet was, and still is, a burning question to scholars. Did it spring from the Egyptian hieroglyphics, or from the cuneiform systems of the Assyrians, or from the hieroglyphics of the Hittites, or from the Syllabary of Cyprus? The *Sefer Yeṣirah* answers: "From the Sefirot."¹⁴ But what does it mean by Sefirot? On this point endless discussions arose; and it has even been disputed whether they are designed to express theological, philosophical, or physical mysteries. Most of the writers, bent on explaining the *Sefer Yeṣirah* in a philosophical way, maintain that by עשר ספירות בלימה, the *Sefer Yeṣirah* meant the so-called Arabic numerals, 1, 2, 3, 4, 5, 6, 7, 8, 9, and 0. This view is shared by Dunash Ibn Tamim,¹⁵ who, however, admits that some people object to it, and maintain that if the *Sefer Yeṣirah* had meant the Arabic numerals, it would have said תשע ספירות (nine Sefirot): for the cipher is not a digit, there are only nine significant numbers. A much weightier reason for opposing the identification of the Sefirot with the Arabic numerals is the fact that the *Sefer Yeṣirah* gives ten as the total number resulting from the addition of the numerals

¹³ The Alphabet, London 1883, I, 125.

¹⁴ See text, § 14.

¹⁵ See *Sefer Yeṣirah*, London 1702, 24, 25.

Dr. H. Malter calls my attention to the following passage of Abraham Abulafia in Jellinek's *הקבלה*, גנזי חכמה הקבלה, Leipzig 1853, p. 20: מוחו של תינוק סופר עשר ספירות שהם נכללין בארבע לפי צורות האצילות בדרך זו א"ב ג"ד (כאלו נכתבו יוד אלפי זה אחר זה). This passage proves that the true meaning of the Sefirot had been known to some Hebrew writers.

1, 2, 3 and 4, and the total of the 9 Arabic numerals added together is 45.

However, there can be no doubt that the Sefirot philosophy of the Sefer Yeṣirah rested on some system of numeral notation. I have studied various systems of antiquity, and I have found that it harmonizes with a numeral system consisting of a series of strokes from one to four, amounting in all to ten | || ||| ||||, and the zero 0. Indeed, the numbers 1, 2, 3, and 4 were originally indicated by such a series of strokes, as is well known to all familiar with the old Roman, Greek, and South-Arabian systems of notation. The numeral systems of the Phœnicians, Egyptians, Babylonians, etc., even indicated the numbers 5, 6, 7, 8 and 9 by the first four groups of strokes, as || ||| = 5, ||| ||| = 6, | ||| ||| = 7, || ||| ||| = 8, and ||| ||| ||| = 9. There can be no doubt, therefore, that by עשר ספירות the Sefer Yeṣirah meant 1, 2, 3, and 4 written in strokes, their number amounting, when added together, to ten, and by בלימה, it meant zero, which, being a symbol for nothing, is the equivalent of בלימה, "nothing" in Hebrew. My belief is that originally the text must have had עשר ספירות ובלימה = ten digits and zero.

As, according to the Sefer Yeṣirah, it is possible to express all numbers by the ten Sefirot,¹⁶ we must demonstrate how all numbers, even those higher than nine, may be indicated by the ten strokes. The strokes to indicate numbers were anciently written vertically and horizontally, as — = ≡ ≡≡. By means of the strokes and the zero, all numbers may be expressed, as they are expressed by the Arabic numerals. The numbers 10, 20, 30, 40, 50 . . . 500

¹⁶ See text, § 16.

...5000....50000 may be expressed as $_o$, $\equiv o$, $\equiv o$,
 $\equiv o$, $\equiv o$, $\equiv oo$, $\equiv ooo$, $\equiv oooo$. 1907 for

instance would be represented thus: $_ \equiv o \equiv$.¹⁷

Such a system of notation is practically the abacus on paper and apparently was the source of the scientific system of notation of the Chinese, and may have been the source of the Arabic numerals, the origin of which is admittedly doubtful. To avoid writing too many strokes, the Chinese made one stroke to represent five units. The numerals 1, 2, 3, 4, 5, 6, 7, 8, 9, 0, were written |, ||, ||| ||||, ||||, 𠄎, 𠄎𠄎, 𠄎𠄎, 𠄎𠄎, 0. 1405536 for instance was written | $\equiv o$ \equiv |||| \equiv 𠄎 (see M. Cantor's *Mathematische Beiträge*, Halle 1863, 47 and 47; first table figures 15, 16).

The East-Arabian figures ١, ٢, ٣, ٤ (1, 2, 3, and 4) originated from the primitive numerals | || ||| ||||. They are only a combination of two, three, and four strokes, to indicate the respective numbers. The figure ٥ for 5 originated from the circle, which in the primitive system of numeral notation was the fifth symbol representing a zero, and in the South-Arabian system of numeral notation was a symbol representing ten units. The East-Arabian figures, ٦, ٧, ٨ (6, 7, and 8) are each a combination of two strokes. The figure ٩ originated apparently from a circle and a stroke, and a dot indicated a zero.

Many writers maintain that only at a later period the principle of position and the zero was discovered. We are, however, now certain that in Babylon, many centuries before the Christian Era, a sexagesimal position was known.

¹⁷ Some Pythagoreans used the strokes with the principle of position without a zero in the columns of an abacus (Dr. M. Cantor's *Mathematische Beiträge*, page 202).

In my opinion, the sexagesimal position originated from the decimal position, and not the reverse; for the earliest abacus, which was doubtless based on a decimal position, is older than the sexagesimal system. F. Cajori says:¹⁸ "The principle of position in its general and systematic application required a symbol for zero. We ask, Did the Babylonians possess one? Had they already taken the gigantic step of representing by a symbol the absence of units?" I am inclined to believe that the zero is as old as the principle of position.¹⁹ The final perfection of the so-called Arabic system of notation consisted, not in the discovery of the principle of position and the zero, but rather in pointing out how the primitive principle of position and the zero can be conveniently used with nine figures, such as 1, 2, 3, 4, 5, 6, 7, 8, and 9.

Although the sexagesimal position was anciently known in Babylonia, yet it was used only, in higher mathe-

¹⁸ A History of Mathematics, New York 1894, p. 7.

¹⁹ In the *American Mathematical Monthly*, 1909, p. 177, G. A. Miller calls attention to the recent change of view in reference to several important questions in the history of elementary mathematics. On page 576 of Cantor's *Vorlesungen über Geschichte der Mathematik* (second edition, vol. I), the following words are found: "According to our opinion the discovery of zero is due to the Hindoos." The corresponding statement in the third edition, page 616, reads as follows: "According to our opinion the discovery is due to the Babylonians, the deepening of the concept is due to the Hindoos." G. A. Miller further says: "The discovery of zero, as used above, implies its use in positional arithmetic. It is certain that the Greeks employed zero in the second century B. C. to denote the absence of degrees, minutes, or seconds in their sexagesimal notation. The earliest known use of this symbol in Babylonian inscriptions belongs to the third century B. C., but it is supposed that it was in use at a much earlier date. At the International Mathematical Congress held in Paris in 1900, Cantor suggested that zero was probably in use among the Babylonians as early as 1700 B. C. Even if such an early date cannot be established, it appears likely that scholars will hereafter attribute the discovery of positional arithmetic to the Babylonians instead of the Hindoos."

matics, as in astronomy, etc.; for ordinary purposes there was a decimal system of notation, without the principle of position. When we find that in Egypt and Phœnicia only a decimal system of notation was used, without the principle of position, it may not be taken as proof that the decimal position and zero were unknown. A decimal system of notation without the principle of position was in ancient times considered more convenient than a decimal system of notation based upon the principle of position.

When it was, that the primitive numerals, |, ||, |||, ||||, o, were changed into the figures used by the East-Arabs, to indicate the numbers 1, 2, 3, 4, 5, 6, 7, 8, and 9, I shall not venture to conjecture. But it is proper to assume, that they may have been known to a few learned men long before they began to be widely used. They may have even been known to the author of the *Sefer Yeşirah*.²⁰ Yet by עשר ספירות בלימה the author meant, not the Arabic numerals, but the primitive numerals, |, ||, |||, ||||, o, from which the Arabic numerals originated.

As, according to the *Sefer Yeşirah*, the alphabet originated from the ten Sefirot, which are, as was demon-

²⁰ As an early Arabian mathematician Mohammed ben Musa Alkharazmi said that the Indians practiced the so-called Arabian system of notation (M. Cantor's *Mathematische Beiträge*, Halle 1863, page 269), arguments have been advanced that the Indians invented this system, or at least taught it to the Arabs. But it is very doubtful what Mohammed ben Musa meant by Indians. By Indians he might have meant Ethiopians, or any other nation under the torrid zone which the ancients used to denominate as Indians (T. Astle, *The Origin and Progress of Writing*, London 1784, p. 41; השלח, IX, 354, 439). Or he could have meant Jewish astronomers like Mashaala who was called by Abraham ibn Ezra an Indian sage (חכם הודו) (see M. Steinschneider, *Arabic Literature of the Jews*, London 1901, p. 107). As the Arabic numeral system of notation apparently originated from the Sefirot philosophy of the *Sefer Yeşirah*, it is proper to share the view of those who claim the Hebrews as its inventors (E. Brooks, *The Philosophy of Arithmetic*, Philadelphia 1876, p. 24).

strated, the first four groups of strokes amounting to ten, we must assume that the first alphabet must have been constructed out of these strokes. This peculiarity harmonizes best with the Libyan-Berberic alphabet, in which we actually find each of the first four groups of strokes: —, =, ≡, ≡≡, |, ||, |||, |||| and the circle ○ or ⊙ is each a letter. Moreover, the other letters of the same alphabet have such forms as point, without a doubt, to their origin in the first four groups of strokes, | || ||| ||||. Two, three, or four strokes are so combined as to indicate the various letters as: T, L, I, — X; □, □, □, □ ×, ×, X; □ □ □ □, etc.²¹

Similar to these characters are the linear letters, which have been found on the Egyptian pottery. On this subject, Mr. W. J. Harding King²² says: "Mr. Evans and Professor Flinders Petrie have shown that certain linear characters which have been found on the Egyptian pottery..... form a signary in which a large number of the characters are identical with the Libyan and Tifinagh. The linear characters in Egypt are earlier than the hieroglyphics, though a few of the forms may ultimately have been fused with the latter. Evolved at a date when hieroglyphic writing was unknown, they persisted with a strange vitality, and were never absorbed or ousted." It is proper to note that, in some Berberic alphabets,²³ one, two, three, or four dots are placed in various positions, to indicate different letters. The four groups of dots, which may be arranged in triangular form .
.
.
., as the

²¹ R. Faulman, *Geschichte der Schrift*, Wien 1880, p. 257. Bickell, *Hebrew Grammar*, Leipzig 1877. Table of Semitic characters by Julius Euting, No. 52, 53.

²² A Search for the Masked Tawareks, London 1903, p. 322.

²³ K. Faulman, p. 257-8.

Pythagoreans arranged them, were originally also symbols representing the first four numbers 1, 2, 3, and 4.²⁴ As already demonstrated, the first alphabet invented was similar to that still used by the Berbers, whence the inference, that there may have been a time when the Hebrews also used such an alphabet. May not כתב ליבונה²⁵ originally have meant Libyan writing?

According to J. Halévy's decipherment of the Libyan alphabet | or — is א, || or = is ו, ||| or ≡ is ע, |||| is ח. According to the *Sefer Yeşirah* (§§ 10, 11, 18, 19, 20, 21) | or — is א, || or = is ו, ||| or ≡ is י, |||| or ≡ is ו. Hence, the vowel-letters אעיו were originally indicated by the four groups amounting to ten strokes, |, ||, |||, ||||, or —, =, ≡, ≡. If these vowel sounds were originally the numeral words for one, two, three, and four, we may assume that the vowel symbols were invented at the same time as the numerals. The invention of such an alphabet as the Libyan must have consisted chiefly of symbols for consonants, because vowel symbols were already in existence from the time the numerals were invented.

In this way we can understand why the vowels (the Tetragrammaton) were originally identical with the ten digits עשר ספירות, | || ||| ||||. This is also in harmony with the following quotations from the commentary 'ה'אוצר:²⁶ "In His great name which is Jehovah" בשמו הגדול הוא "for He comprises the Ten Sefirot." כי הוא הכולל לכל י. "When it mentions the 'mothers' (vowels) it means the Seifirot themselves." כשזוכר אמות מדבר בעצמי הספירות

²⁴ Cyrus Thomas in the 19th Annual Report of the Bureau of American Ethnology, Smithsonian Institution, 1897-98, p. 860.

²⁵ Sanhedrin 21b.

²⁶ *Sefer Yeşirah*, Warsaw 1884, p. 69, 74, 90.

"It was explained in the second chapter that, wherever it says 'mothers,' it alludes to the Sefirot themselves." כְּבִר פִּירֵשׁ בַּפ"ב כִּי כָל מָקוֹם שֶׁאָמַר אִמּוֹת יִרְמוֹז לְעֲצֵמֵי הַסְּפִירוֹת.

The four elements, air, water, fire, and earth, from which it was anciently believed everything was created, emanated, according to the Sefer Yeširah, from the vowels אֵי; air emanated from the א, water from the ע, fire from the ו, and earth from the י. As the vowels were originally identical with the ten Sefirot, it is from the Sefirot that the whole universe emanated.²⁷ The Sefirot cosmogony is given by the Sefer Yeširah as follows:

"With 'one,' the living God of the Universe graved and hewed out voice, air, and speech, and this is the Holy Spirit. With 'two' God graved and hewed out void and chaos. Void is a green line that surrounds the whole universe, and chaos refers to viscous stones, sunk in the abyss, whence water comes forth."

"With 'three' God graved and hewed out mud and clay. He arranged them like a garden bed. He set them up like a wall. He covered them like a pavement, and poured upon them water, and the earth was formed."

"With 'four' God graved and hewed out the Throne of Glory, the ophanim, the seraphim, the holy animals, and the ministering angels."²⁸

Chaos signified in the ancient cosmogonies the vacant, infinite space out of which sprang all things that exist. Later cosmogonists, such as Ovid, represent it as the confused, shapless mass, out of which the universe was formed into a cosmos, or harmonious order. W. Enfield²⁹

²⁷ See text, § 17.

²⁸ See text, §§ 18-21.

²⁹ History of Philosophy, London 1819, I, 180.

says: "By Chaos some writers understood water, and make this the first material principle." He further says:³⁰ "The theogonists certainly do not suppose God to have been prior in the order of time to matter: they speak of Chaos as eternal, and seem to have been wholly unacquainted with the doctrine of creation from nothing." This is at variance with the *Sefer Yeşirah*, which emphasizes the doctrine of creation from nothing by the statement: "He (God) formed existence out of void, something out of nothing" (*Mishnah* 22). It holds that chaos was not even the first thing created, but was preceded by voice, air, and speech.

The numeral system of the *Sefer Yeşirah*, as I have explained, may be considered to consist of five symbols, |, ||, |||, ||||, o, the cipher being the fifth symbol. From this symbol originated the numeral o (5) in the East-Arabian notation. It is identical in form with the letter • (ה) of the Arabic alphabet. As the ה indicates the absence of a vowel letter, and is similar in its nature to the zero, which indicates the absence of a digit, it is possible that the ה originated from the zero. The first primitive numerals |, ||, |||, ||||, o, were thus primarily identical with the five vowel-letters אעויה which were originally the letters of the Tetragrammaton יהוה. The five primitive symbols are the five elements with which God created the universe, the ה or the zero being the fifth element, as the *Sefer Yeşirah* says:³¹ יצר מתהו ממש ועשה אינו ישנו וחצב אבנים גדולות מאויר שאינו נחפש וסימן לדבר עשרים ושתיים מנין ורוה אחר.

"He formed existence out of void, something out of nothing, and He hewed large stones out of intangible air, thus, twenty-two in number, one in spirit."

³⁰ *Ibid.*, 131.

³¹ See text, § 1 f.

The word תהו (void) has the same meaning as בלימה; it is also equivalent to zero, which symbolizes the creation of something out of nothing. This is in accordance with the teaching of the Pythagoreans who counted the void surrounding the universe as a fifth element (Wilhelm Bauer, *Der ältere Pythagoreismus*, Berne 1897, pp. 83, 84, 88, 89).

We must conclude that the so-called Arabic numerals and the alphabet originated from the ten digits and the zero, or rather from two symbols, 1 0, the stroke and the circle. L. D. Nelme,³² in his essay on the origin of letters, shows that all elementary characters, or letters, derive their forms from the line and the circle. As I understand the Sefer Yeširah, it also holds that all written characters originated from a line and a circle, but from a line that was originally a symbol for unity, and a circle that was originally the symbol for zero. Similarly, all cuneiform characters originated from two symbols 𐎶 𐎵, those for one and ten.

L. L. Conant³³ says: "Two centuries ago the distinguished philosopher and mathematician Leibnitz proposed a binary system of numeration, the only symbols needed in such a system would be 0 and 1 . . . Leibnitz found in the representation of all numerals by means of two digits 0 and 1 a fitting symbolization of the Creation out of chaos or nothing, of the Universe by the power of the Deity." We have seen that not only a binary system of numeration, but even the decimal system may be expressed by a stroke and a zero. Moreover, it has been

³² Comp. *Edinburgh Encyclopaedia*, I, p. 540. A copy of this work, which the present writer saw in the New York Public Library, had on the title page, 'L. D. Nelme, London, 1762.'

³³ The Number Concept, New York 1896, p. 102.

pointed out that the alphabet and the so-called Arabic numerals originated from these two symbols. Therefore, the author of the *Sefer Yeşirah* may have meant by two —, with which God created void and chaos, a digit and a zero; for as the ten digits may be expressed by nine digits and a zero, so may two digits be represented by a digit and a zero. Thus, the *Sefer Yeşirah* may have believed two digits, 0 and |, a fitting symbolization of the creation, out of Chaos or nothing, of the universe, by the power of the Deity.

Because the zero in the number ten occupies the space of a digit, only nine strokes, not ten, are used; and, therefore, we may say that the numeral system of the *Sefer Yeşirah*, consists of nine digits and the zero. For | || ||| |||| 0 = ||||| ||||| 0, and the symbol ✱ expresses the whole numeral system. The latter figure was anciently known in South-Arabia, and it represented, according to Halévy, an ע.³⁴ The *Sefer Yeşirah* calls the letters “stones” (אבנים), because they originated from the numerals (Sefirot), which were originally indicated by stones. It is possible, therefore, that the ✱ is the philosopher’s stone, from which the Arabic numerals, the alphabet, and all civilization originated.


Moreover, since in ✱ are united the ten Sefirot, the vowels, and the Tetragrammaton, it apparently symbolized the angel containing the name יהוה referred to in Exodus 23, 20. 21. This angel is chiefly known in kabbalistic literature by the name מַטְטְרוֹן שֶׁר הַפָּנִים “Metatron, Prince of the Face” and is identified with the prophet Elijah.³⁵

³⁴ Comp. D. H. Müller, *Epigraphische Denkmäler aus Arabien*, Vienna 1889, p. 19.

³⁵ ילקוט ראובני, Lemberg 1860, p. 47b.

He is the *שר היצירה* "Prince of Creation," or the "Logos," with which God created the Universe. Upon the crown of the head of this angel "The Holy one—Blessed be He" wrote letters with which were created Heaven, earth, seas, rivers, etc., and all the elements of creation.³⁶ To this angel God intrusted all the secrets of the Law, and of wisdom; and all the mysteries of creation are known to him as they are known to the Creator Himself.³⁷ As the Arabic numeral system of notation, the vowels, and the Tetragrammaton were originally identical, they all contain the same 'gretat secret' the revelation of which is forbidden by the author of the *Sefer Yeṣirah* (text, §§ 10, 11, 14). This secret apparently is the angel

אלהו הנביא = מטטרון שר הפנים

symbolized by the letter *ע*, which was anciently written also as .

The most mysterious character in Jewish history is the prophet Elijah. It was he who demonstrated that *יהוה* is the true God and no other. He is called the angel of the Covenant, *מלאך הברית*. He is believed to be present at the circumcision. It is also believed that he is bound to come and decide all knotty points in the law, and to appear before the true Messiah arrives.

In kabbalistic literature, not only the vowel letters, but all the letters are considered images of angels. In *Sefer Raziel* (ed. Amsterdam, p. 126) occurs the following:

כאדם שחקק אותיות מן דמות אותם המלאכים שנטרו ונער בהם
הקב"ה והפילים משמי מרום ושכח את דמותם מאלף ועד תי"ו

"Like Adam who engraved letters out of the likeness of those angels who had been driven away, whom the

³⁶ *Ibid.*, 52b.

³⁷ *Ibid.*, 48b.

Holy One, Blessed be He, rebuked and cast down from the high heavens. He then graved their likeness from Aleph to Taw."

The vowel letters, however, the letters of the Tetragrammaton, are images and pictures of superior angels. Since the cipher η is also a letter of the Tetragrammaton, it is also a picture and image of a superior angel. The angel which the cipher η symbolizes, is apparently Satan שטן , who appears before God to accuse all mortal beings as he accused Job and Joshua son of Jehozadak (Job 1, 6-12; 2, 1-7; Zechar. 3, 1). Like the cipher η so the zero in the Arabic system of notation, and in the system from which the latter originated, was anciently believed to symbolize the evil deity, or Satan,³⁸ in contrast to the numeral one 1, which was thought to symbolize the good God, from whom all good emanated. The numeral one 1 and the zero 0 are a fitting symbolization of the two opposing principles concerning which the Sefer Yesirah (§ 23) says as follows:

"Also God set the one over against the other, good against evil, and evil against good; good out of good; and evil out of evil; good testing evil, and evil testing good; good is stored away for the good, and evil is stored away for the evil."

³⁸ "It was not very easy to comprehend at first the precise force of the cipher, which, insignificant by itself, only serves to determine the rank and value of the other digits. A sort of mystery, which has imprinted its trace on language, seemed to hang over the practice of numeration, for we still speak of deciphering, and of writing in cipher, in allusion to some dark or concealed art" (The Philosophy of Arithmetic, by John Leslie, Edinburgh 1817, p. 114). "Indeed, in the early history of arithmetic in Europe the system was regarded as belonging to black art and the devil; and it was, no doubt, this popular prejudice that delayed its general introduction into Christian Europe" (E. Brooks, The Philosophy of Arithmetic, Philadelphia 1901, p. 107).

The account of the origin of letters and numerals the author of the Sefer Yeşirah concludes as follows:

“When Abraham our father arose, he looked and saw and investigated and observed and engraved and hewed and combined and formed and calculated, and his creation was successful. Then the Master of all revealed himself to him, and made a covenant with him and with his seed forever. He made a covenant with him on the ten fingers of his hands, and this is the covenant of the tongue; and on the ten toes of his feet, and this is the covenant of circumcision; and tied the twenty-two letters of the “Torah” to his tongue and revealed to him their secret . . .”

The name of Abraham which is mentioned in the closing Mishnah suggested to many commentators that Abraham himself wrote the Sefer Yeşirah. As such a view is entirely repugnant to the modern critical mind, some writers regard this closing Mishnah as a later interpolation. In my opinion there is no doubt that this Mishnah belongs to the original Sefer Yeşirah, for it is in perfect harmony with all the original material. The mention of the name of Abraham does not indicate that the Patriarch wrote the Sefer Yeşirah, but that he was the inventor of the alphabet, the scribe (סופר) mentioned in the opening Mishnah as the person whom God inspired with it.

It is worthy of note that Philo attributes the first invention of letters to Abraham (Rees, *Cyclopaedia*, art. *Letter*; comp. also Suidas in Abraham and Isidor Hispal, Örigg, I. 3).

(*To be continued*)